Look Up At the Sky and Count the Stars If You Can

Infinity, Mathematics, and God

2017 Kenneth C. Simpson Essay Prize
Sheil Catholic Center at Northwestern University
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In Galileo’s words, “mathematics is the language in which God wrote the universe.” What could have motivated such a statement? In fact, Galileo is continuing the tradition of St. Augustine of Hippo, who looked to Biblical passages such as Isaiah 40:26, “see who has created these: He leads out their army and numbers them, calling them all by name” as an indication that God used numbers in the creation of the world (Is. 40:26, NAB).[1] A connection between mathematics and God, whether positive or negative, may appear dubious at first. In the so-called culture wars of today, whether real or invented, a completely blind faith is in an inevitable conflict with a completely objective science. Yet pure mathematics and theology are often left out of the discussion. In contemporary culture, math and theology may seem like two entirely different kinds of abstractions, each with its separate premises and forms of logical thinking. However, some of the preeminent theologians and mathematicians of history believe that the two disciplines are deeply related. To illustrate this connection, I first introduce infinity and its connection to faith, then explore these concepts through a historical analysis of how Christians develop an understanding of the relationships between infinity and God.

A multitude of Biblical passages, both in the Hebrew Scriptures and in the New Testament, make claims similar to Isaiah 40:28: “The Lord is the eternal God, creator of the ends of the earth. He does not faint nor grow weary, and his knowledge is beyond scrutiny” (Isa. 40:28, NAB). How can this be? How can something eternal, omniscient, and omnipotent even exist, much less descend to earth and take human form? Infinity in math is similar to eternity, which I define as infinity with respect to time, and to omniscience and omnipotence, which are

[1] Adam Drozdek, “Beyond Infinity: Augustine and Cantor,” *Laval théologique et philosophique* Vol 51, no. 1 (1995): 127-40.

infinity of knowledge and power. Through math, we understand that infinity not only exists, but is also a logical and rational concept that we can use in theorems and derivations. Even without fully grasping the meaning of infinity, we can still discuss it in a meaningful way and form relevant conclusions. Take, for example, the classic problem about a firm deciding between two options: A) to cooperate with another firm and earn a small amount ($10) now and each period in the future, or B) to fail to cooperate and earn a larger sum ($20) now, but earn zero in all future periods because the other firm punishes it for failing to cooperate. The total value of B is clearly $20. But what is the value of A? According to basic financial principles, a dollar tomorrow is worth less than a dollar today because a dollar today can be invested to have more than one dollar tomorrow. The value of a dollar tomorrow is $1 times some fraction ∂. Thus the value of A is $10 today plus 10\partial$ tomorrow plus 10\partial^2$ the next day, and so on for an infinite number of periods:

$$\text{Value of A} = \$10 + \$10\partial + \$10\partial^2 + \$10\partial^3 + \ldots.$$

Mathematically, this infinite series is equal to $10/(1-$\partial$). Without a notion of infinity, it is more difficult to compare option A and option B. How many periods in the future would we receive a payment in A? How would we make this decision? Yet using infinity, the math operates perfectly and we have numerical payoffs for A and B which we can compare to make a decision. We can still use these ideas and make this decision even though we cannot precisely grasp what it means for a series to continue without end.

Another classic example of the cruciality of infinity in mathematics is the Law of Large Numbers. Roughly speaking, the law holds that as an event occurs an infinite number of times, the observed probability will match the expected probability. When I flip a coin, we theoretically expect heads and tails to happen with equal probability, that is, to occur an equal number of times. However, if I flip a coin only ten times, it is highly likely that there will be six heads and four tails or seven tails and three heads, for example, instead of five heads and five tails. If I flip a coin twenty or thirty or one hundred times, the fraction of heads will likely be closer and closer to one half. It is only as I flip the coin an infinite number of times that the fraction of heads is exactly one half, but we cannot guarantee a one-half ratio with any finite number of coin flips. Statistics relies upon the Law of Large Numbers to estimate probabilities of events occurring from a sample of data. Through asymptotic justification, or proofs that assume the number of data points "approaches infinity," advanced mathematics as well as research studies in the sciences and social sciences draw their conclusions. For example, statistics might use the Law of Large Numbers and the idea of infinity to identify the probability that a particular method of medical treatment is effective.

Mathematics thus teaches us how to comprehend infinity as much as we can and teaches us that it is an inherently rational concept which has meaning even if we do not fully understand it. If we understand that infinity exists and entire branches of mathematics in fact rely upon it, then the concept of a God having existed forever in the past and existing forever into the future seems more possible. God and eternity can exist, even if we cannot fully understand them. After all, as St. Augustine reminds us, "if you understand it, it is not God." [2] If someone begins with the mathematical framework of infinity, a God who is "the one who is and who was and who is to come" seems possible, not like the being of some myth or magic (Rev. 1:8, NAB). When we hear the Scripture passage "for God so loved the world that he gave his only Son, so that everyone who believes in him might not perish but have eternal life" we at least know that eternity, if not eternal life, is real (John 3:16, NAB).

[2] "Si comprehendis, non est Deus" (Augustine, "Sermones 52").

While today infinity is mathematically accepted and we can use our understanding of infinity to better understand and appreciate God, the concept of infinity was not accepted in the world of the early Christians or even the late medieval era. Therefore, early Christian mathematicians used their understanding of God to develop the concept of infinity, a crucial contribution to math and the sciences. Developments in the fields of mathematics and theology are closely intertwined and mutually beneficial to each other. In order to better understand this trajectory, I begin the historical overview with Aristotle's *Physics*, which forms the framework for pre-Christian thought on infinity.

Aristotle postulates that while potential infinity can exist in some circumstances, actual infinity cannot. He writes in *Physics* that space must be finite and that "time and movement are indeed unlimited, but only as processes, and we cannot even suppose their successive stretches to exist." [3] His stance allows potential infinity but states clearly that "infinity cannot exist as an actualized entity and as substance or principle." [4] For instance, Aristotle thinks that a line can potentially extend forever in either direction, but cannot do so in reality. While this distinction may seem insignificant to a modern reader, it leads Aristotle to conclude that any god must be finite. This restriction on infinity, and his overall emphasis on finiteness, not only limited the characteristics of a god but also the development of science and mathematics.[5]

Early Christian theologians break this barrier of finiteness and potential infinity by arguing that God is infinite, enabling new developments in math and the sciences. Their faith in an omniscient and omnipotent God with power over even death, pointing toward the existence of an infinite entity, helps inspire and motivate their theories. After all, it is difficult to reconcile

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[3] Aristotle, *Physics*, trans. Philip H. Wicksteed/Francis M. Cornford (Cambridge, Mass./London), III.208a20
[4] Ibid. III. 204a21
[5] Wolfgang Achtner, "Infinity in Science and Religion: The Creative Role of Thinking about Infinity," *Neue Zeitschrift Für Systematische Theologie Und Religionsphilosophie*, Vol. 47, no. 4 (2005): 392-411.

omniscience and omnipotence with Aristotle's emphasis on the finite. Gregory of Nyssa not only claims that God is infinite but also "stresses the infinite way of the ascending soul to the infinite God." [6] Since humans are finite and imperfect, and thus cannot reach God or complete virtue, our path toward God is infinite. Because God is infinite and real, Gregory and theologians such as St. Augustine firmly establish that actual infinity, not just potential infinity, exists.

While St. Augustine's view is slightly different, it amplifies the concept of infinity rather than diminishes it. For Augustine, although God is not finite or infinite, he understands infinity and extends beyond it. In Aristotle's philosophy, the sequence of integers {... -2, -1, 0, 1, 2, 3…} is able to go on without end (potential infinity), but we cannot grasp it as a complete sequence because we cannot reach the beginning or end (it is not actual infinity). Augustine surmounts this perspective by claiming that, since God is all-powerful, "it pleases the divine mind, entirely unchanging, whose knowledge of the infinite and of countless things [exists] without the method of numerical cognition." [7] Thus, roughly speaking, this sequence is able to be understood by God to have a beginning and an end, and therefore actual infinity exists. This particular justification for infinity would be nonsensical without Augustine's firm belief in an all-powerful God. Here, faith clearly motivates a key mathematical development. Furthermore, his definition of God as beyond infinity inspires later mathematicians, such as Georg Cantor, to consider levels of infinity, as will be discussed later.

Yet Augustine also emphasizes the contributions that mathematical progress makes to theology. In his view, each of us has some innate understanding of infinity--without this understanding, we could not identify what is finite. For instance, imagine a circle with two radii drawn so near each other that they almost seem to touch. A line has no width, and thus we know

[6] Wolfgang Achtner, "Infinity in Science and Religion: The Creative Role of Thinking about Infinity," 392-411.
[7] "Mentem divinam omnino immutabilem cuius libet infinitatis capacem et innumera omnia sine cogitationis alternatione numerantem" (Augustine, *De civitate Dei* XII.17).

that these two lines cannot in fact be touching each other. Because the radii are so close together, we may not be able to physically draw a line between them which does not touch either radii. However, Augustine explains, "reason proclaims that innumerable such can be drawn, which, in these incredibly narrow spaces, can come into no contact with each other except at the centre." [8] It is impossible for us to derive this fact from pure observation of the geometric figures—we cannot physically draw even one more radii between the two existing radii that does not touch the others. Thus, we must have some inborn concept of infinity within ourselves which allows us to understand geometry as well as other finite objects and quantities. In Augustine's view, this innate knowledge of infinity is tied to his conception of an "inner light," given by God, which each human has within oneself, whether or not he or she knows God. [9] We presuppose that infinity exists and then come to a greater--but never full--understanding of it through reason, particularly through math. Whether God is considered infinite or beyond infinity, some appreciation of infinity is necessary or at least very helpful for learning about God.[10] Today, the English version of the Catechism of the Catholic Church uses the infinite almost thirty times in discussion of God's attributes and works, the nature of truth, and differences between humanity and God.[11] Given this importance, for Augustine, mathematics is crucial for coming to know God.[12] Furthermore, since "even the ungodly understand eternity" exploring infinity and eternity, which is infinity with respect to time, through mathematics can be a way toward God.[13]

While theologians are clear that we can never fully comprehend infinity, and thus can never fully comprehend God, what is the nature of this gap? How can we appreciate what we

[8] Augustine, *Soliloquies,* trans. Rose Elizabeth Cleveland (Boston, Little, Brown, and Company, 1910).
[9] Drozdek, "Beyond Infinity: Augustine and Cantor," 127-40.
[10] Ibid.
[11] Catholic Church, *Catechism of the Catholic Church*, 2nd ed. (Vatican, Libreria Editrice Vaticana, 2000).
[12] Drozdek, "Beyond Infinity: Augustine and Cantor," 127-40.
[13] "Nam hinc est quod etiam impii cogitant aeternitatem" (Augustine of Hippo, *De Trinitate*, 14.21).

cannot fully grasp? While Gregory of Nyssa provides a transcendental explanation, described by Achtner as “the infinite way of the ascending soul to the infinite God,” Nicolas of Cusa provides a symbolic and intellectual explanation using mathematics. [14] For Nicolas, the infinity of God is the “coincidencia oppositorum,” or the coming together of opposites, and he develops a way of symbolizing infinity rationally with geometry.[15] He claimed that in infinity, a line, a triangle, and a circle come together even though they are in many ways opposites. In the circle example, focus on a segment of the circle, and then expand the diameter of the circle infinitely. The circle then becomes an infinite line. [16] The triangle illustration is more complicated but nonetheless illustrates infinity using logical thought. In this approximation of infinity using finite geometry, Nicolas refutes the apostates who claim that we can only discuss God in terms of what He is not. He successfully defends the role of reason in our faith in God.[17] The importance of this contribution is difficult to overstate; theology, famously defined by St. Anselm as faith seeking understanding, inherently employs our reason and logic.

Furthermore, Nicolas’ definition of truth, which uses infinity, can be especially meaningful in the present day. Nicolas’ formulation also help to explain St. Thomas Aquinas’ definition: “truth is the equation of things and the intellect.” [18] In today’s world of relativism, it is often claimed that perfect truth cannot exist because we do not see it achieved concretely in anything around us--not in other people, nature, or even in science, with its openness to disproving current theory. In our current culture, we may be tempted to believe that there are many correct ways to interpret morality--many truths. That is, we may doubt that “things” and intellect can be equated in one true way because we cannot completely see that way. By arguing

[14] Wolfgang Achtner, "Infinity in Science and Religion: The Creative Role of Thinking about Infinity," 392-411.
[15] Nicolas of Cusa, *De Docta Ignorantia,* trans. Jasper Hopkins (A.J. Benning, 1996).
[16] Ibid. I.13.35-41.
[17] Wolfgang Achtner, "Infinity in Science and Religion: The Creative Role of Thinking about Infinity," 392-411.
[18] “Veritas est adaequatio rei et intellectus” (Thomas Aquinas, *Summa Theologiae,* 1.16.1).

that truth is attainable only by an infinite number of steps, Nicolas explains how absolute truth can exist even if we cannot fully perceive it in the world around us. An infinite process is required to equate the “things” which we see and our intellect. Since humans are only finite, we do not possess absolute truth; only an infinite or beyond infinite God can fully possess it.

As Nicolas describes,

> It is self-evident that there is no comparative relation of the infinite to the finite …Therefore, it is not the case that by means of likeness a finite intellect can precisely attain the truth about things…The intellect is to truth as an inscribed polygon is to the inscribing circle. The more angles the inscribed polygon has the more similar it is to the circle. However, if the number of its angles is increased ad infinitum, the polygon never becomes equal [to the circle] unless it is resolved into an identity with the circle.[19]

Although Nicolas did not himself use the branch of mathematics known as set theory, it can illustrate the “self-evident” first claim here. Suppose we have an infinite set {1,2,3,4,5…} and a finite set {1,2}. If we “subtract” the finite set from the infinite set, we have the set {3,4,5…}, which is still an infinite set. “Subtracting” the set {1,2} does not make the set {1,2,3,4,5....} any smaller, and {3,4,5…} is equally as much “larger” than {1,2} as {1,2,3,4,5…} is “larger” than {1,2}. In order to describe a true size relationship between {1,2} and {1,2,3,4,5…}, the difference in size between {1,2} and {1,2,3,4,5…} must be larger than the difference in size between {1,2} and {3,4,5…}. Because the difference is the same, we cannot describe a true “size” relationship between {1,2} as {1,2,3,4,5…}. This is precisely because {1,2,3,4,5…} and {3,4,5…} are infinite and {1,2} is finite. Thus, we cannot truly discuss the comparative “size” or “comparative relation” between the infinite and the finite.

[19] Nicolas of Cusa, *De Docta Ignorantia,* I.3.10

As Nicolas continues, this idea helps explains why human intellect, which is finite, cannot relate perfectly to truth, which is infinite. His geometric image of a polygon inscribed in a circle provides concreteness to a highly abstract idea, and math is once again helpful in providing approximations for theology (Brient).[20] Picturing the polygon approaching the circle, we have an (imperfect) metaphor for our own personal journeys toward truth and God. In sum, Nicolas' definition shows us that truth, just like infinity, cannot be perfectly seen on earth but yet is real, logical, and relevant to us.

Building upon his faith, Georg Cantor (1845-1918) brought new precision to mathematical infinity by developing the modern theory of sets. A devout Christian who corresponded extensively with Catholic theologians and clergy, including Pope Leo XIII, Cantor's study of mathematics grew from a strong religious motivation. He was especially influenced by Pope Leo XIII, who discussed the Neo-Thomist theory that immorality was a consequence of incorrect philosophy. In this school of thought, false beliefs about the natural world (including mathematics) resulted in atheism and materialism.[21] Thus, correct study of nature is of great use to the Church, and scholastic theories could help science and mathematics avoid factual errors, thus reducing later moral errors. Pope Leo XIII thus advocates in his theories encyclical *Aeterni Patris*:

> For, the investigation of facts and the contemplation of nature is not alone sufficient for their profitable exercise and advance; but, when facts have been established, it is necessary to rise and apply ourselves to the study of the nature of corporeal things, to

[20] Elizabeth Brient, "How Can the Infinite Be the Measure of the Finite? Three Mathematical Metaphors from De Docta Ignorantia," in *Cusanus: A Legacy of Learned Ignorance,* ed. Peter J. Casarella, Elizabeth Bryant, and Louis K. Dupre (Catholic University of America Press, May 2014), 210-225.

[21] Joseph W. Dauben, "Georg Cantor and Pope Leo XIII: Mathematics, Theology, and the Infinite," *Journal of the History of Ideas* Vol. 38, no. 1 (1977): 85–108.

> inquire into the laws which govern them and the principles whence their order and varied unity and mutual attraction in diversity arise.[22]

This encyclical and other theological writings inspired Cantor to go to such great lengths to develop his theories regarding infinity.

Cantor, like Augustine, believed that infinity was an inborn concept that was necessary to perceive the world.[23] He quotes extensively from Augustine's *De civitate Dei* in his 1888 work "Mitteilungen," including the rather strong statement that "they who speak against those things which are infinite can neither understand God nor science."[24] [25] As once again mathematicians are involved in a debate about the existence of infinity, Cantor's religious certainty that infinity must exist helps propel his mathematical proofs of this fact.[26] Further developments in math since the eras of Augustine and Nicolas had brought new doubts about infinity. While today we may think of the existence of infinity as obvious, it is revealing to note that even Galileo, Gottfried Wilhelm Leibniz, Baruch Spinoza, and Isaac Newton deny its existence because they believed it led to logical inconsistencies.[27] With proofs, especially of this complexity, first knowing the result with certainty makes it easier to devise the structure and elements of the proof. Additionally, Cantor faces intense criticism for this theory and others, not only by mathematicians but also by theologians who think that his theories either diminished the power of God or supported heretical beliefs.[28] His faith both prevents him from recanting and propels him to convince other mathematicians, theologians, and the Catholic Church of this truth.[29] The

[22] Leo XII, *Encyclical Letter, Aeterni Patris*, The Holy See, 4 August 1879. Web. May, 2013.
[23] Drozdek, "Beyond Infinity: Augustine and Cantor," 127-40 .
[24] Georg Cantor, "Mitteilungen zur Lehre vom Transfiniten" (Pfeffer, 1887).
[25] "Contra eos, qui dicunt ea, quae infinita sunt, nec Dei posse scientia comprehendi" (Augustine, *De civitate Dei*, XII.19).
[26] Joseph W. Dauben, "Georg Cantor and Pope Leo XIII: Mathematics, Theology, and the Infinite," 85–108.
[27] Wolfgang Achtner, "Infinity in Science and Religion: The Creative Role of Thinking about Infinity," 392-411.
[28] Dauben, "Georg Cantor and Pope Leo XIII: Mathematics, Theology, and the Infinite," .
[29] Ibid.

end result is a victory over finite mathematics for the modern era, leaving a crucial impact on further development of mathematics. Indeed, it is difficult to imagine the state of mathematics today if discourse were limited to the finite.

In his proofs for the existence of infinity, Cantor resolves certain supposed logical inconsistencies with infinity. Following in the intellectual tradition of Augustine, Cantor believed that God was beyond infinity, a belief which became crucial for solving a supposed paradox. The existence of a God who fully comprehends infinity but yet is beyond it presents the possibility of an entity beyond infinity and asks the question of whether infinity can have multiple levels. Cantor then develops a theory of different types and levels of infinity. The first distinction was infinities of different sizes, or "cardinalities." The smallest type of infinity is the countable infinity, and an example is the set of natural numbers N={1,2,3,4…}, with cardinality defined as $\aleph_0$. Cantor conceptualized $\aleph_0$ to be a number in itself, as will be discussed later. Another example of a set which Cantor showed to have the cardinality $\aleph_0$ is the set of all integers, Z={...,-2,-1,0,1,2…}.[30] To understand the cardinality of these sets, consider the larger infinite set R, the set of all real numbers, which notably includes integers, fractions, and numbers like Π. While we can imagine counting off N or Z, listing their numbers in some sort of order, it is impossible to do the same for R. Imagine trying to list the numbers in R starting from 0: 0, .000000001--yet there are numbers between 0 and .000000001 or between 0 and .0000000000001 or…so the task is clearly impossible. Thus we can understand that R is in some sense larger than N and Z, and R is a different "level" of infinity.

[30] Richard Hammack, *Book of Proof (*Richmond, VA, 2013), 219.

To understand sets larger than R and to ultimately understand the paradoxes Cantor resolves, it is necessary to first define what it means for a set to contain another set. Suppose set A={1,2,3}, set B={9,10,11}, and set C={17,18,19}. Then we can define a set D={A, B, C}={ {1,2,3}, {9,10,11}, {17,18,19} }, where D contains the sets A, B, and C. A subset of B is a set that contains only elements from B. For example, E={9}, F={10,11}, and G={9,10,11} are all subsets of B. The power set of B, denoted P(B), is the set that contains all subsets of B. In this example, P(B)={ {9}, {10}, {11}, {9,10}, {9,11}, {9,10,11} }. Returning to the idea of a set "larger" than R, we can say that P(R), or the power set of R, which contains all subsets of R, has a higher cardinality, or "size," than R. Thus, P(R) is a new "level" of infinity, and P(P(R))), the power set of the power set of R, which contains all subsets of the power set of R, is yet even greater and is another "level" of infinity. According to Cantor, the power set of a set always has a greater cardinality than the set itself.[31] Now think about the set of all sets, ח, which contains N, Z, R, and every other set. Because P(ח) is the power set of ח,

Cardinality P(ח) > Cardinality ח

However, since ח contains all sets, it must contain P(ח), the power set of ח. Thus we also have

Cardinality ח > Cardinality P(ח)

This is clearly a mathematical contradiction. Therefore, the set of all sets ח cannot be a mathematical concept--it is beyond mathematics and the natural world. Cantor identifies this crucial distinction, and labeles infinite sets like N, Z, and R as the transfinite, or the infinity of the world and in mathematics, and labeled ח as the Absolute infinity, which is only for God.[32] Cantor's Christianity is integral in forming this idea, in conceiving of an infinity that is beyond even mathematics, and resolving a paradox that that could have disproved the existence of

[31] Wolfgang Achtner, "Infinity in Science and Religion: The Creative Role of Thinking about Infinity," 392-411.
[32] Adam Drozdek, "Beyond Infinity: Augustine and Cantor," 127-40 .

infinity itself. Because God is omniscient, he must be able to grasp all sets, including the set of all sets ת. Cantor further explains,

> the transfinite with its profusion of transformations and forms by necessity points to an Absolute, to a 'true infinity,' whose magnitude can be neither augmented nor diminished and which, therefore, is considered quantitatively, is an absolute maximum.[33]

For example, the transfinite numbers include $\aleph_0$, which is the cardinality of sets like N or Z, and the cardinalities of other infinite sets. His exact proof of the existence of transfinite numbers relies on the existence of irrational numbers, which were not yet fully accepted at the time. With this new mathematical precision, Cantor gives clarity to the idea that the existence of infinity points toward God. Earlier theologians and mathematicians successfully argue that we, as finite, could not invent the infinite, therefore a higher power must exist to have created infinity. Cantor, however, illustrates more concretely the levels of infinity and the possibility--and likelihood--of a God existing beyond that. Furthermore, because immutability—as an absolute maximum it cannot be made larger or smaller—is an essential property of the Absolute, it is also a useful symbol of God's immutability. While some theologians and mathematicians of his time claim that belief in the transfinite is belief in a false power, a false god, he firmly believed that the existence of transfinite numbers show that God is even more powerful. If God has created that which is infinite, then how great must God be?

Cantor's theories pose the question of the Continuum Hypothesis, a powerful example of what we cannot know about mathematics. Roughly speaking, the continuum hypothesis proposes that no set A exists that has a cardinality between $\aleph_0$ (the cardinality of N) and the cardinality of R, that is

[33] Georg Cantor, *Gesammelte Abhandlungen* (Hildesheim, Georg Olms, 1962), 405.

$$\aleph_0 < |A| < |R|$$

where the bars around the set indicated cardinality of the set. [34] Furthermore, Kurt Godel and Paul Cohen's work proves that the Continuum Hypothesis cannot be proven or disproven.[35] Treating the Hypothesis as true or as false leads to two different branches of mathematics, both of which are completely valid logically.[36] As this example demonstrates, while some parts of math are completely certain, other aspects are not only unknowable, but we have proven that we cannot know them. This stands in contrast to the sciences, where phenomena that we do not understand are simply phenomena we do not understand yet. Realizing, even proving, that we cannot know a mathematical idea brings a sense of humility, a recognition that we are not all-knowing or all-powerful. And if we cannot know everything even in a field as certain as math, then what we cannot know about God does not diminish the certainty of our faith.

In the proofs of Nicolas of Cusa, Georg Cantor, and each of the mathematicians of history and today, an unusual form of certainty exists. Unlike in the sciences, where we never can be completely certain of what we know, some areas of math offer complete certainty. Mathematical laws are true in the infinite sense. That is, they hold regardless of location and time and are unchangeable. Yet they are relevant to our individual lives. In this way, mathematical laws are mere hints at the characteristics of God--hints at omnipresence, eternity, and omnipotence, yet meaningful for individuals. In mathematics, then, we can see God, in a different way than we can see God in nature or in those around us. Furthermore, mathematics can help reassure us that absolute truth exists, even if people have not always believed all elements of it at all times--the idea of infinity governed us no less when even the greatest scholars doubted its existence.

[34] Richard Hammack, *Book of Proof*, 237.
[35] Ibid.
[36] Ibid.

Genesis describes how, when God made his covenant with Abraham, “He took him outside and said, ‘Look up at the sky and count the stars, if you can. Just so,’ he added, shall your descendants be.’ Abram put his faith in the Lord” (Gen. 15:5, NAB). Can we count the stars? Are they a countably infinite set, just like the natural numbers? God uses the innumerable and infinite number of the stars to illustrate the magnitude of the covenant and his blessing to Abraham. The concept of infinity, too, is part of our inheritance and blessings from God, along with the rationality that enables us to discover and discuss it. The study of infinity points our minds toward the existence of the heavens and helps us glimpse an eternal God we cannot fully understand. Although we are finite, we know that God “numbers all the stars, calls each of them by name” (Ps. 147:4-5, NAB). As this discussion of infinity shows, scholars throughout history not only put their faith in God, but employ mathematics to help deepen their faith and build upon their faith to make advances in mathematics.

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